BUAD 300
Dr. Robinson
Some Review Questions to Aid in Preparation for First Exam

Write the correct formula first, then place the numerical data into the formula and solve. Use only pencil and eraser. Do not use programmable calculators.

1) Given the information below, what are E(X), E(Y), σ_X^2, σ_Y^2, $\sigma_{X,Y}$?

State j	X_j	Y_j	Prob j
1	9	11	¼
2	10	10	½
3	11	9	¼

$$E(X) = \sum_i X_i P_i = 9\left(\tfrac{1}{4}\right) + 10\left(\tfrac{1}{2}\right) + 11\left(\tfrac{1}{4}\right) = 10$$

$$E(Y) = \sum_i Y_i P_i = 11\left(\tfrac{1}{4}\right) + 10\left(\tfrac{1}{2}\right) + 9\left(\tfrac{1}{4}\right) = 10$$

$$\sigma_X^2 = \sum_i (X_i - E(X))^2 P_i = (9-10)^2\left(\tfrac{1}{4}\right) + (10-10)^2\left(\tfrac{1}{2}\right) + (11-10)^2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}$$

$$\sigma_Y^2 = \sum_i (Y_i - E(Y))^2 P_i = (11-10)^2\left(\tfrac{1}{4}\right) + (10-10)^2\left(\tfrac{1}{2}\right) + (9-10)^2\left(\tfrac{1}{4}\right) = \tfrac{1}{2}$$

$$\sigma_{X,Y} = \sum_i (X_i - E(X))(Y_i - E(Y)) P_i = (9-10)(11-10)\left(\tfrac{1}{4}\right) + (10-10)(10-10)\left(\tfrac{1}{2}\right) + (11-10)(9-10)\left(\tfrac{1}{4}\right) = -\tfrac{1}{2}$$

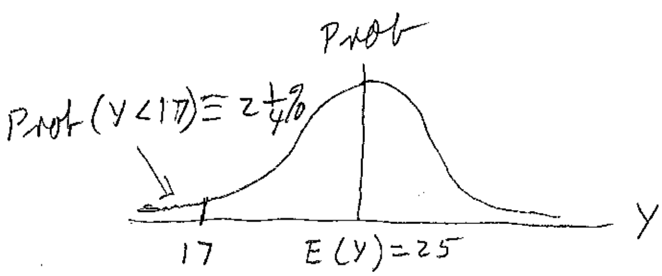
2) X is normally distributed. If Y = 5 + 4X and if E(X) = 5, and $\sigma_X^2 = 1$, what is E(Y) and σ_Y^2? What is the probability of Y < 17?

$$E(Y) = 5 + 4E(X) = 5 + 4(5) = 25$$

$$\sigma_Y^2 = 4^2 \sigma_X^2 = 16(1) = 16 \rightsquigarrow \sigma_Y = 4$$

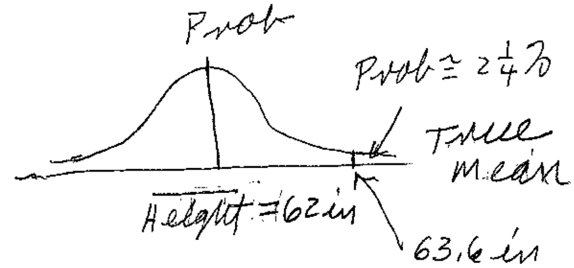
$$Z_{calc} = \frac{E(Y) - HYP\,Y}{\sigma_Y} = \frac{25-17}{4} = 2$$

Prob $(Y < 17) \cong 2\tfrac{1}{4}\%$



3) A sample of 25 of the Fredonia State students has a mean height of 62 inches, and a standard deviation of 4 inches. What is the probability of H_0: The true mean height < 63.6 inches? Test H_0 at a significance level of 95%? $SE = \frac{S}{\sqrt{n}} = \frac{4}{\sqrt{25}} = \frac{4}{5}$

$t_{calculated} = \frac{\text{True Height} - \overline{\text{Height}}}{SE} = \frac{63.6\text{ in} - 62\text{ in}}{4/5\text{ in}} = 5\left(\frac{1.6}{4}\right) = 2$



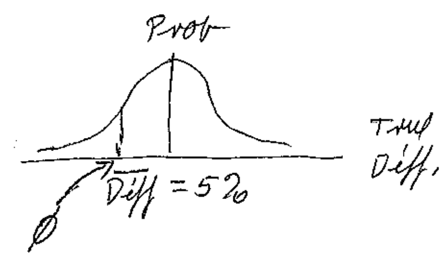
Prob(Mean height < 63.6 in.) ≈ $97\frac{3}{4}$ %

Decision rule concerning H_0:
If Prob($\text{True}_{\text{Mean}}$ < 63.6 in) ≥ 95%
then do not reject H_0.

Decision:
Do not reject H_0.

4) You have data from two samples of students, 30 males and 30 females, concerning their scores on an English essay. The mean score for the males is 75%, and the standard error is 3. The mean score for the females is 80%, and the standard error is 4. What is the probability of H_0: The true mean score for females > the true mean score of males? Test H_0 at a significance level of 95%. The degrees of freedom for this t statistic are 58. Diff = Mean female − Mean male

$t_{calculated} = \frac{\overline{\text{Mean females}} - \overline{\text{Mean males}}}{\sqrt{SE_M^2 + SE_F^2}} = \frac{80\% - 75\%}{\sqrt{3^2+4^2}} = \frac{5\%}{5\%} = 1$



Prob(Mean of females > Mean of males) ≈ 83%

Decision rule: If Prob($\text{True}_{\text{Diff}}$ > ∅) ≅ 83%

Decision: Do not accept H_0!

5) (a) Given the information below, what is $\rho_{X,Y}$?

$\sigma_{X,Y} = -2,\ \sigma_X^2 = 1,\ \sigma_Y^2 = 4,\ E(X) = 9,\ E(Y) = 30$

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = -\frac{2}{(1)(2)} = -1$$

(b) What is the range for any possible ρ?

$$-1 \le \rho \le +1$$

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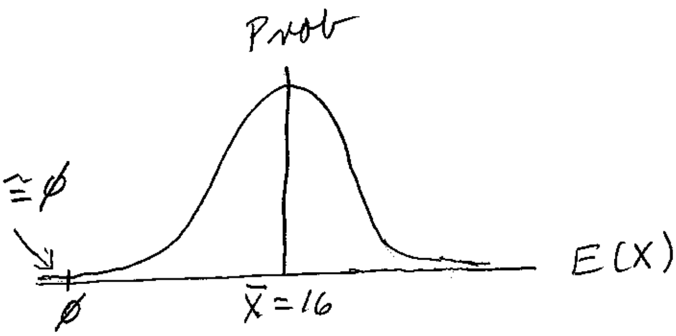
Some Example Questions and Solutions

1) A sample of 16 from a normally distributed population indicates $\bar{x} = 16$ and $\frac{s}{\sqrt{n}} = 4$. What is Prob (E(X) < 0)?

$$t_{calc} = \frac{\bar{x} - \text{hyp } E(X)}{s/\sqrt{n}} = \frac{16 - \emptyset}{4} = 4$$

$Prob\,(E(X) < \emptyset) \cong \emptyset\%$



2) You wish to test the following hypothesis about the normally distributed variable X.

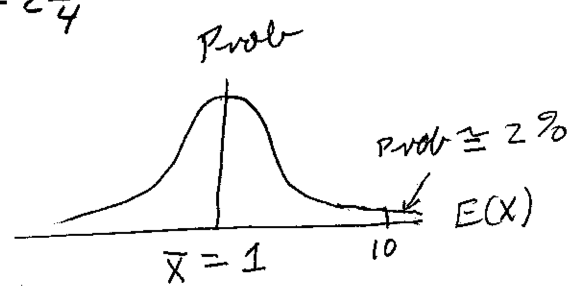
H_0: E(X) ≥ 10

A sample of 26 indicates that the sample mean $\bar{x} = 1$, and $\frac{s}{\sqrt{n}} = 4$. Do you reject H_0 at a confidence level of 95%?

$$t_{calc} = \frac{\bar{x} - \text{Hyp } E(X)}{s/\sqrt{n}} = \frac{1 - 10}{4} = -2\tfrac{1}{4}$$

$d.f. = n-1 = 26-1 = 25$

$t_{critical} = 1.708$
$\alpha = 5\%$



Decision rule: If $t_{calc} \geq t_{critical}$, then do not reject H_0.

Decision: Reject H_0!

or Decision rule: If $Prob(E(X) \geq 10) \geq 95\%$, then do not reject H_0.

Decision: Reject H_0.

For this problem, what is $t_{critical}$ if the sample size is very large? $t_{critical} = z_{critical} = 1.645$

3) X is normally distributed. A very large sample has a mean of 100, and a standard error of 10. Compute a 95% confidence interval for the true population mean (E(X))?

95% confidence interval = sample mean $\pm$ $z_{critical}$ $s/\sqrt{n}$

$\alpha/2 = 2\frac{1}{2}\%$

$= 100 \pm 1.96(10)$

$= 100 \pm 19.6 = \begin{cases} 119.6 \\ 80.4 \end{cases}$

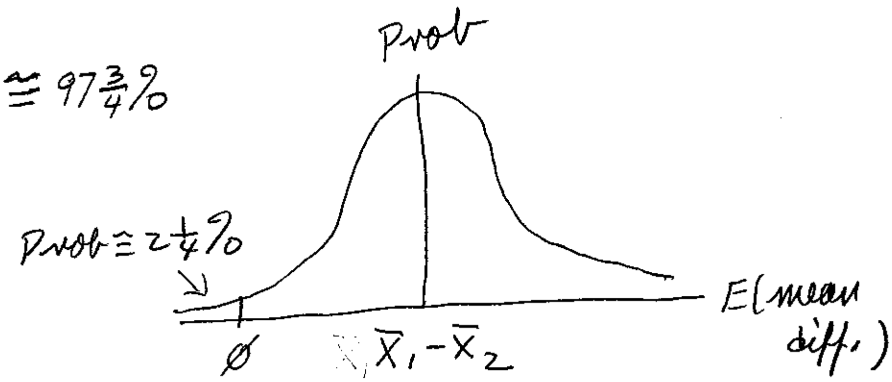
4) For very large samples, $\overline{X_1} = 100$, and $\overline{X_2} = 90$. The standard error for sample 1 is 3, and for sample 2 it is 4. Using a confidence level of 95%, test the hypothesis H_0: The true mean of population 1 exceeds that of population 2? Assume the samples are from normally distributed populations. $t_{critical} = z_{critical} = 1.645$

$$t_{calc} = \frac{\overline{X}_1 - \overline{X}_2 - D}{\sqrt{SE_1^2 + SE_2^2}} = \frac{100 - 90 - \emptyset}{\sqrt{3^2 + 4^2}} = \frac{10}{5} = 2$$

Decision rule: If $t_{calc} \geq t_{critical}$, then do not reject H_0.

Decision: Do not reject H_0.

Note: Prob(E(mean diff.) $> \emptyset$) $\cong 97\frac{3}{4}\%$



5) For very large samples, $\overline{X_1} = 100$, and $\overline{X_2} = 80$. The standard error for sample 1 is 3, and for sample 2 it is 4. Using a confidence level of 95%, test the hypothesis H_0: The true mean of population 1 exceeds that of population 2 by 15? Assume the samples are from normally distributed populations.

$$t_{critical} = z_{critical} = 1.645$$

$$t_{calc} = \frac{\overline{X}_1 - \overline{X}_2 - D}{\sqrt{SE_1^2 + SE_2^2}} = \frac{100 - 80 - 15}{\sqrt{3^2 + 4^2}} = 1$$

Decision rule: If $t_{calc} \geq t_{critical}$, then do not reject H_0.

Decision: Reject H_0.

Note: $Prob(\overline{X}_1 - \overline{X}_2 - 15) \cong 84\%$