BUAD 300
Prof. Robinson
Review Questions for OLS:

1) For model equation (1) above, and for the information below, what is R^2, and what is ρ?

$\sigma_Y^2 = 2$, $\sigma_e^2 = 2$

$$Y_t = \beta_0 + \beta_1 X_t + e_t \quad \Big\} \text{ model equation}$$

$$R^2 = 1 - \frac{\sigma_e^2}{\sigma_y^2} = 1 - \frac{2}{2} = \emptyset$$

$$\rho_{x,y} = \sqrt{R^2} = \emptyset$$

2) Given the information below, and given model equation (1), what is β_1? What is β_0? What is $\rho_{X,Y}$? What is R^2?

$\sigma_{X,Y} = 1$, $\sigma_X^2 = 4$, $\sigma_Y^2 = 4$, , $\overline{Y} = 100$, $\overline{X} = 80$

$$\beta_1 = \frac{\sigma_{x,y}}{\sigma_x^2} = \frac{1}{4}$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X} = 100 - \frac{1}{4}(80) = 80$$

$$\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y} = \frac{1}{(2)(2)} = \frac{1}{4}$$

$$R^2 = \rho_{x,y}^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

BUAD 300
Dr. R. Robinson
Review Questions: The second exam will be on Thursday, April 18.

Write the correct formula first, then place the numerical data into the formula, and solve. Use proper grammar. Use pencil and eraser only. Do not use a programmable calculator.

1) Given the model below, and the variance – covariance matrix (see explanation at end of page 5), what are the OLS estimates for β_1 and β_2? Given the expected values (or means for each variable) what is β_0?

$W = \beta_0 + \beta_1 X + \beta_2 Y + \in$ where X and Y and $\in$ are all random, with $\in$ being the residual error.
Variance – covariance matrix

	Y	X	W
Y	1	4	6
X	4	4	1
W	6	1	8

E(Y) = 20 E(X)= 20 E(W) = 2

$\beta_1 = \frac{\sigma_{X,W}}{\sigma_X^2} = \frac{1}{4}$

$\beta_2 = \frac{\sigma_{Y,W}}{\sigma_Y^2} = \frac{6}{1}$

$\beta_0 = \overline{\cancel{Y}X} - \beta_1 \bar{X} - \beta_2 \bar{Y}$
$= 2 - \frac{1}{4}(20) - 6(20)$
$= -123$

If Var($\in$) = 1, then what is the R^2 for this regression?

2) (a) What does "OLS" stand for? Briefly explain the meaning of "L" and "S"?

(b) What are the OLS assumptions for $\in$ in the above model?

(c)What is the OLS assumption for X and Y in the model equation above (in question #1)?

3) Given the information below, formally test the following hypotheses. Use α = 5% for all of these tests.

(i) H_0: $\beta_1 > 0$? $t_{calc} = \frac{\text{OLS } \beta_1 - \text{HYP. } \beta_1}{SE} = \frac{3 - \emptyset}{2} = 1\frac{1}{2}$

$t_{critical} \approx$ 1.645

Decision rule: If $t_{calc} \geq t_{critical}$ do not reject H_0.

Decision: Reject H_0.

(ii) H_0: $\beta_1 = \beta_2 = 0$?

$F_{calculated} =$ 10

Associated p-value = < 5%

Decision rule: If P-value < 5% then reject Ho!

Decision: Reject Ho!

(ii) Fill in the F and p values in the table below?

Predictor	Coef	S.E. of Coef.
β_0	4	1
β_1	3	2

$R^2 = .90$

(State p > 5% or p < 5%.)

Analysis of Variance

Source	DF	SS	MS	F	p
Regression	2	200	$\frac{200}{2}$	$\frac{100}{10} = 10$	<5%
Residual Error	47	470	$470/47$		
Total	49	670			

4) You compare OLS estimates of models (1) and (2) as given below. The *SS Residuals* for model (1) is 80. The *SS Residuals* for model (2) is 60. Test the hypothesis, "H_0: Model (2) has greater explanatory power than model (1)." n = 30.

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \qquad (1)$$

$$Y_t = \beta_0 + \beta_1 X_t + \beta W_t + \varepsilon_t \qquad (2)$$

$F_{calculated} = \frac{SSE_1 - SSE_2}{SSE_2}(n-3) = \frac{80-60}{60}(27) = 9$

$F_{critical} \simeq 5$

Decision rule: If $F_{calc} \geq F_{critical}$ then do not reject Ho!

Decision: Do not reject Ho!

5) Test the hypothesis, "H_0: The errors in your OLS estimation are normally distributed." Your $\chi^2_{calc} = 5$. What is $\chi^2_{critical}$ for $\alpha = 5\%$ and 10 cells? Fill in the following:

$\chi^2_{critical} = 14.07$

Decision rule: If $\chi^2_{calc} < \chi^2_{critical}$ then do not reject H_0!

Decision: Do not reject H_0!

6) An OLS regression with 60 observations has 30 negative residuals. You count 22 runs. If E(u) = 31, and Var(u) = 16 what is the $z_{calculated}$? What is $z_{critical}$? Can you reject the hypothesis, "H_0 The residuals are random." Let α = 5%.

$z_{calculated} = \frac{u - E(u)}{\sigma_u} = \frac{22-31}{4} = \frac{9}{4} = 2\frac{1}{4}$

$z_{critical}$ = 1.96 since this is a two-tailed test.

Decision rule: If $|z_{calc}| > z_{critical}$ then reject H_0!

Decision: Reject H_0!

7) You calculate the following models (1) and (2) for casinos A and B by OLS regression.

Coin In_t A = $\beta_0 + \beta_1 Promo_t + u_t$ (1)

Coin In_t B = $\beta_0 + \beta_1 Promo_t + u_t$ (2)

Given the regression output below, test the hypothesis, "H_0: β_1 for A > β_1 for B." Use α = 5% and d.f. ≅ 198 for your test.

OLS for A

	Coef	S.E. of Coef
β_0	100	50
β_1	20	3

$t_{calc} = \frac{\text{OLS } \beta_1 \text{ for A} - \text{OLS } \beta_1 \text{ for B}}{\sqrt{SE_A^2 + SE_B^2}} = \frac{20+2}{\sqrt{3^2+4^2}} = \frac{22}{5} = 4\frac{2}{5}$

OLS for B

	Coef	S.E. of Coef
β_0	100	50
β_1	-2	4

$t_{critical} \approx$ 1.645 for a one-tailed test.

Decision rule: If $t_{calc} \geq t$ critical then do not reject H_0!

Decision: Do not reject H_0!

8)You estimate an OLS regression for model (1) below. The results are also given below. Is β_1 significantly different from 0? Is β_2 significantly different from 0? Sample is large!

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 W_t + \varepsilon_t \qquad (1)$$

<u>OLS for A</u>

	Coef	S.E. of Coef
β_0	100	50
β_1	20	10
β_2	-10	10

H_0: β_1 is <u>significantly different from</u> 0.

$t_{calculated} = \dfrac{OLS\ \beta_1 - Hyp.\ \beta_1}{SE_{\beta_1}} = \dfrac{20 - \emptyset}{10} = 2$

$t_{critical} \approx$ 1.96 for a two-tailed test.

Decision rule: If $|t_{calc}| \geq t_{critical}$ then do not reject H_0.

Decision: Do not reject H_0!

H_0: β_2 is <u>significantly different from</u> 0.

$t_{calculated} = \dfrac{OLS\ \beta_2 - HYP.\ \beta_2}{SE} = \dfrac{-10 - \emptyset}{10} = -1$

$t_{critical} =$ 1.96 for a two-tailed test.

Decision rule: If $|t_{calc}| \geq t$ critical then do not reject H_0.

Decision: Reject H_0!

8) An OLS regression estimate of model (1) is shown below. What are the mean average coin-in values for each day of the week?

$$\text{Coin-in}_t = \beta_0 + m(0 \text{ or } 1) + u(0 \text{ or } 1) + w(0 \text{ or } 1) + r(0 \text{ or } 1) + f(0 \text{ or } 1) + s(0 \text{ or } 1) + \varepsilon_t \qquad (1)$$

$$= 10 - 2(0 \text{ or } 1) - 1(0 \text{ or } 1) + 1(0 \text{ or } 1) + 2(0 \text{ or } 1) + 3(0 \text{ or } 1) + 4(0 \text{ or } 1) + \varepsilon_t$$

Sunday mean = 10

Monday mean = 10 − 2 = 8

⋮

Saturday mean = 10 + 4 = 14

9) You estimate the regression model $W = \beta_0 + \beta_1 X + \beta_2 Y + \varepsilon$. Given the information below, what is σ_W^2? X, Y, and $\in$ are independent.

$\beta_0 = 5,\ \beta_1 = 3,\ \beta_2 = -4,\ \sigma_X^2 = 1,\ \sigma_Y = 2,\ \sigma_\varepsilon^2 = 2$

$$\sigma_w^2 = \beta_1^2\sigma_X^2 + \beta_2^2\sigma_Y^2 + \sigma_\varepsilon^2 = 3^2(1) + (-4)^2(2) + 2$$

$$= 9 + 32 + 2 = 43$$

Note: A variance-covariance matrix is of the form below.

	X	Y	W
X	σ_X^2	$\sigma_{X,Y}$	$\sigma_{X,W}$
Y		σ_Y^2	$\sigma_{Y,W}$
W			σ_W^2