

Where relevant, write the correct formula first, then place the numerical data into the formula and solve. Be neat and clear. Use 95% probability significance for all hypothesis tests. Always specify both the critical value and calculated value of the test statistic, the decision rule, and the decision. Rankings and other symbols can be placed alongside the data below.

1) Twenty consumers tested both products X and Y. Their preferences are listed below. Test the following H_0 : Consumers prefer brand Y. Use the sign test.

Consumer j	Brand Preference	Consumer j	Brand Preference
1	Y	11	Y
2	Y	12	Y
3	X	13	Y
4	Y	14	Y
5	Y	15	X
6	Y	16	X
7	X	17	Y
8	Y	18	Y
9	X	19	Y
10	Y	20	Y

$n = 20$

$\#OY_0 = 15$

$E(\#OY_0) = \frac{1}{2}(n) = 10$

$\sigma = 2.07/10$

$Z_{calc} = \frac{\#OY_0 - E(Y)}{\sigma_Y}$

$= \frac{15 - 10}{2.07/10} = \frac{50}{20}$

$= 2.5 \sqrt{20} \sqrt{20}$

$= 2.5 (19) \sqrt{5}$

$= 5 \sqrt{5}$

≈ 11.18

$Z_{calc} \approx 12$

Calculated test statistic:

Critical value of the test statistic:
 $Z_{critical} = 1.645$ for a one-tailed test.

Decision rule:

If $t_{critical} \geq t_{actual}$ do not reject H_0 .

Decision:

Do not reject H_0 .

2) Eleven students tested two routes from a particular parking space to a particular classroom, with each student walking each route. The times are recorded below. Test the following H_0 : The two routes are equally fast? Use a signed-rank test.

Student j	Route 1	Route 2	Diff	Rank	Signed Rank
1	10.2 minutes	10.1 minutes	+0.1	1	+1
2	9.6	9.8	-0.2	2	-2
3	9.2	8.9	+0.3	3	+3
4	10.6	10.2	+0.4	4	+4
5	9.9	10.4	-0.5	5	-5
6	10.2	9.6	+0.6	6	+6
7	10.6	9.9	+0.7	7	+7
8	10.0	10.0	0.0		
9	11.2	10.4	+0.8	8	+8
10	10.7	9.8	+0.9	9	+9
11	10.6	9.6	+1.0	10	+10

$T = 48 - 7 = 41$

$\sigma_T = \sqrt{10(11)(21)/6} = 19.62$

Calculated test statistic: $Z_{calc} = \frac{T - E(T)}{\sigma_T} = \frac{41 - 0}{19.62} = 2.09$

Critical value of the test statistic: $Z_{critical} = 1.96$ for a two-tailed test.

Decision rule: If $|Z_{calc}| > Z_{critical}$ then reject H_0 .

Decision: Reject H_0 !

3) The overall percentages-of-total-possible-credit in a particular statistics class were recorded for marketing majors, management majors, accounting majors and finance majors. The rankings of the data are presented below. Test the following H_0 : The majors have equivalent scores? Use the Kruskal-Wallis test.

Marketing	Management	Accounting	Finance
1	2	3	4
5	6	7	8
12	11	10	9
16	15	14	13
21	22	23	24
20	19	18	17
<u>75</u>	<u>75</u>	<u>75</u>	<u>75</u>

$$H = \sum \frac{R_i^2}{n_i} - 3(n_T + 1) = \frac{12}{24(25)} 4 \left(\frac{9}{75^2} \right) - 3(25) = \phi =$$

Calculated test statistic:

Critical value of the test statistic: $\chi^2_{critical} = 7$

Decision rule: If $H \geq \chi^2_{critical}$ then reject H_0 !

Decision: Do not reject H_0 !

4) The overall percentages-of-total-possible-credit in a particular statistics class were recorded for 20 students along with the students' age in months. The rankings of the data are given below. Test the following H_0 : Age is unrelated to performance in this class? Use rank correlation.

Student j	Age	%	D_1	D_2
1	1	1	0	0
2	2	2	0	0
3	4	3	+1	1
4	3	4	-1	1
5	5	7	-2	4
6	6	5	+1	1
7	7	6	+1	1
8	8	8	0	0
9	9	9	0	0
10	10	10	0	0

$$r = 1 - \frac{6 \sum D_i^2}{n(n^2-1)} = 1 - \frac{6(8)}{10(99)} = 1 - \frac{48}{990} \approx 1 - .05 \approx .95$$

$$\rho = \left(\frac{n-1}{3} \right)^{\frac{1}{2}} = \frac{1}{3}$$

Calculated test statistic: $Z_{calc} = \frac{r - \rho}{\frac{1}{\sqrt{3}}} = \frac{.95 - \frac{1}{3}}{\frac{1}{\sqrt{3}}} = 2.85$

Critical value of the test statistic: $Z_{critical} = 1.96$ for a two-tailed test.

Decision rule: If $|Z_{calc}| \geq Z_{critical}$ then reject H_0 !

Decision: Reject H_0 !

5) A sample of 15 graduating students, 10 females and 5 males, was randomly selected. Their GPAs were recorded, and the ranks of these GPAs for the pooled sample are presented below. Test the following hypothesis: H_0 : There is no difference in GPAs between the genders? Use the Mann-Whitney test.

10	}	6	}	$N_1 = 5$
9		46		
8		T = 46		
7		3		
15		12		
5		11		
4		14		
13		12		
2		11		
1		6		
Females		Males		

$N_2 = 10$

$$E(T) = N_1 \left[\frac{N_1 + N_2 + 1}{2} \right] = 5 \left(\frac{16}{2} \right) = 40$$

$$\sigma_T^2 = \frac{N_1 N_2 (N_1 + N_2 + 1)}{12} = \frac{5(10)(16)}{12} = 66.67$$

$$\sigma_T = 8.16$$

Calculated test statistic: $Z_{cal} = \frac{T - E(T)}{\sigma_T} = \frac{46 - 40}{8.16} = 0.74$

Critical value of the test statistic: $Z_{critical} = 1.96$ for a two-tailed test.

Decision rule: If $|Z_{cal}| \geq Z_{critical}$ then reject H_0 .

Decision: 00 not reject H_0 !

$$E(T) = n_1 \left[\frac{n_1 + n_2 + 1}{2} \right]$$

$$\sigma^2 = \frac{n_2 n_1 (n_1 + n_2 + 1)}{12}$$

$$r = 1 - \frac{6 \sum d_i^2}{n(n_2 - 1)}$$

$$d_i = x_i - y_i$$

$$\sigma_r = \sqrt{1/(n-1)}$$

$$H = \left\{ \frac{12}{n(n_2 + 1)} \sum R_i^2 \right\} - 3(n_2 + 1) \quad \text{d.f.} = k-1 \quad k \text{ is the number of samples.}$$

$$E(\mu_r) = 0$$

$$\sigma_r = \sqrt{\frac{n(n+1)(2n+1)}{6}}$$

$$E(\mu) = n/2$$

$$\sigma = n/2$$

6) Define perfect duty? Define imperfect duty? Which of these duties has practical limits? Present two statistical examples of imperfect duty in statistics?