Note that I will give you the diagram on Page 3.

BUAD 300
Dr. Robinson
First Exam Example

Write the correct formula first, then place the numerical data into the formula and solve. Use only pencil and eraser. Do not use programmable calculators.

1) Given the information below, what are E(X), E(Y), σ_X^2, σ_Y^2, $\sigma_{X,Y}$?

State j	X_j	Y_j	Prob j
1	8	11	¼
2	10	10	½
3	12	9	¼

$$E(X) = \sum_i X_i P_i = 8(\tfrac{1}{4}) + 10(\tfrac{1}{2}) + 12(\tfrac{1}{4}) = 10$$

$$E(Y) = \sum_i Y_i P_i = 11(\tfrac{1}{4}) + 10(\tfrac{1}{2}) + 9(\tfrac{1}{4}) = 10$$

$$\sigma_X^2 = \sum_i (X_i - E(X))^2 P_i = (8-10)^2(\tfrac{1}{4}) + (10-10)^2(\tfrac{1}{2}) + (12-10)^2(\tfrac{1}{4}) = 2$$

$$\sigma_Y^2 = \sum_i (Y_i - E(Y))^2 P_i = (11-10)^2(\tfrac{1}{4}) + (10-10)^2(\tfrac{1}{2}) + (9-10)^2(\tfrac{1}{4}) = \tfrac{1}{2}$$

$$\sigma_{X,Y} = \sum_i (X_i - E(X))(Y_i - E(Y)) P_i$$

$$= (8-10)(11-10)(\tfrac{1}{4}) + (10-10)(10-10)(\tfrac{1}{2}) + (12-10)(9-10)(\tfrac{1}{4})$$

$$= -\tfrac{2}{4} + 0 - \tfrac{2}{4} = -1$$

2) X is normally distributed. If Y = 3 + 6X and if E(X) = 5, and $\sigma_X^2 = 1$, what is E(Y) and σ_Y^2? What is the probability of Y < 17?

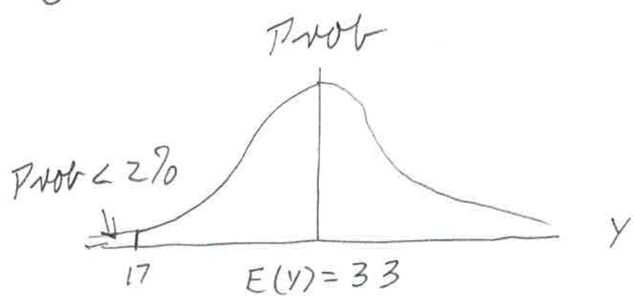
$$E(Y) = 3 + 6E(X) = 33$$

$$\sigma_Y^2 = 6^2 \sigma_X^2 = 36(1) = 36$$

$$\sigma_Y = 6$$

$$z_{calc} = \frac{E(Y) - Hyp.(Y)}{\sigma_Y} = \frac{33-17}{6} = \frac{16}{6} = 2\tfrac{4}{6} = 2\tfrac{2}{3}$$

Prob (Y < 17) < 2%



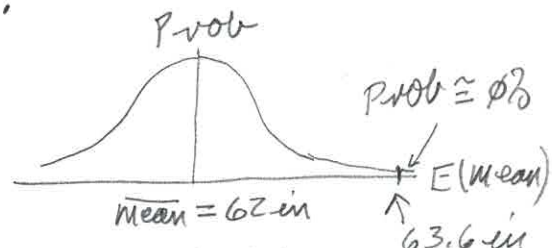
3) A sample of 100 of the Fredonia State students has a mean height of 62 inches, and a standard deviation of 4 inches. What is the probability of H_0: The true mean height < 63.6 inches? Test H_0 at a significance level of 95%?

$$t_{calculated} = \frac{\text{HYP. } E(\text{mean}) - \text{sample mean}}{s/\sqrt{n}} = \frac{63.6 \text{ in} - 62 \text{ in}}{4/10} = \frac{10\,(1.6 \text{ in})}{4 \text{ in}} = 4$$

Prob(Mean height < 63.6 in.) ≈ 99.9%

Decision rule concerning H_0: If Prob (E(mean) < 63.6 in) ≥ 95%, then do not reject H_0.

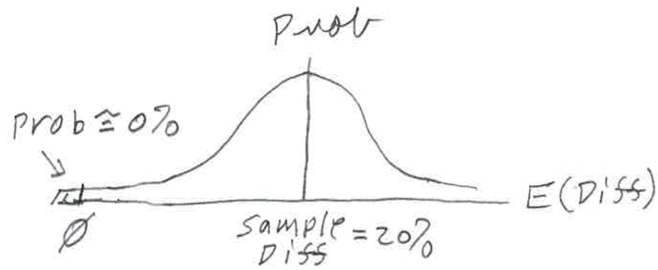
Decision: Reject H_0.



4) You have data from two samples of students, 30 males and 30 females, concerning their scores on an English essay. The mean score for the males is 60%, and the standard error is 3. The mean score for the females is 80%, and the standard error is 4. What is the probability of H_0: The true mean score for females > the true mean score of males? Test H_0 at a significance level of 95%. The degrees of freedom for this t statistic are 58. Diff = mean females − mean males

$$t_{calculated} = \frac{\text{sample mean females} - \text{sample mean males}}{\sqrt{SE_f^2 + SE_m^2}} = \frac{80\% - 60\%}{\sqrt{3\%^2 + 4\%^2}} = \frac{20\%}{5\%} = 4$$

Prob(Mean of females > Mean of males) ≈ 99.99%



Decision rule:
If Prob (E(Diff) > Ø) ≥ 95% then do not reject H_0.

Decision:
Do not reject H_0.

5) (a) Given the information below, what is $\rho_{X,Y}$?
$\sigma_{X,Y} = -3$, $\sigma_X^2 = 4$, $\sigma_Y^2 = 4$, $E(X) = 9$, $E(Y) = 30$

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = -\frac{3}{(2)(2)} = -\frac{3}{4}$$

(b) What is the range for any possible ρ?

$-1 \leq \rho \leq +1$