

Example Second Exam: Write the correct formula first, then place the numerical data into the formula and solve. Use “=” signs, and use proper math. Use only pencil and eraser. Do not use programmable calculators. Use the front and back of these sheets of paper. Each question is worth 15 points.

1) (a) What does “OLS” stand for? Explain the meaning of “L” and “S?”

(b) What are the OLS assumptions for the residual error in a regression?

(c) What is the OLS assumption for the independent variables in a regression?

2) Given the model below, and the variance – covariance matrix, what are the OLS estimates for β_1 and β_2 ? Given the expected values (or means for each variable) what is β_0 ?

$Y = \beta_0 + \beta_1 X + \beta_2 W + \epsilon$ where X and W and ϵ are all random, with ϵ being the residual error.

Variance – covariance matrix

	Y	X	W
Y	13	4	8
X	4	4	1
W	8	1	8

$$\beta_1 = \frac{\sigma_{X,Y}}{\sigma_X^2} = \frac{4}{4} = 1 \quad \left\{ \quad \beta_2 = \frac{\sigma_{W,Y}}{\sigma_W^2} = \frac{8}{8} = 1 \right.$$

$$\begin{aligned} \beta_0 &= E(Y) - \beta_1 E(X) - \beta_2 E(W) \\ &= 50 - 1(10) - 1(10) = 30 \end{aligned}$$

$$R^2 = 1 - \frac{\sigma_\epsilon^2}{\sigma_Y^2} = 1 - \frac{1}{13} = \frac{12}{13}$$

If $\text{Var}(\epsilon) = 1$, then what is the R^2 for this regression? Answer with a fraction.

- 3) An OLS regression with 60 observations has 30 negative residuals. You count 20 runs. If $E(u) = 32$, and $\text{Var}(u) = 16$ what is the $z_{\text{calculated}}$? What is z_{critical} ? Can you reject H_0 : The errors are random? ($\alpha = 5\%$)

$$\sigma_u = \sqrt{\text{Var}(u)} = \sqrt{16} = 4$$

$$z_{\text{calculated}} = \frac{u - E(u)}{\sigma_u} = \frac{20 - 32}{4} = -3$$

$$z_{\text{critical}} = 1.96 \text{ because this is a two-tailed test.}$$

Decision rule: If $|z_{\text{calc}}| > z_{\text{critical}}$ then reject H_0 .

Decision: Reject H_0 !

- 4) You compare OLS estimates of models (1) and (2) as given below. The *SS Residuals* for model (1) is 80. The *SS Residuals* for model (2) is 50. Test the hypothesis H_0 : model (2) has greater explanatory power than model (1)? $n = 28$, $\alpha = 5\%$.

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \quad (1)$$

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 W_t + \varepsilon_t \quad (2)$$

$$F_{\text{calculated}} = \frac{SSE_1 - SSE_2}{SSE_2} \left(\frac{n-3}{1} \right) = \frac{80-50}{50} (25) = \frac{3}{5} (25) = 15$$

$$F_{\text{critical}} \approx 4.24; \text{ d.f. are } n-3 = 25$$

Decision rule: If $F_{\text{calc}} > F_{\text{critical}}$ then do not reject H_0 .

Decision: Do not reject H_0 !

5) Given the information below, formally test the following hypotheses: (Use $\alpha = 5\%$ for all of these tests.)

$$d.f. = 39 - 2 = 37$$

(i) $H_0: \beta_1 > 0?$

$$t_{\text{calculated}} = \frac{\text{OLS } \beta_1 - \text{HYP. } \beta_1}{\text{SE of } \beta_1} = \frac{3 - 0}{3} = 1$$

$$t_{\text{critical}} \approx 1.645$$

Decision rule: If $t_{\text{calc}} > t_{\text{critical}}$ then do not reject H_0 .

Decision: Reject $H_0!$

(ii) $H_0: \beta_1 = \beta_2 = 0?$

$$F_{\text{calculated}} = 10$$

$$F_{\text{critical}} \approx 3.25$$

Decision rule: If $F_{\text{calc}} > F_{\text{critical}}$ then reject H_0 .

Decision: Reject $H_0!$

(ii) Fill in the F and p values in the table below? For the p-value, you only need to indicate if $p > 95\%$ or $p < 95\%$.

Predictor	Coef	S.E. of Coef.
β_0	4	1
β_1	3	3
β_2	2	1
$R^2 = .90$		

Analysis of Variance

Source	DF	SS	MS
Regression	2	200	200/2
Residual Error	37	370	370/37
Total	39	570	

Indicate either < 95% or > 95%.

$F = 100/10 = 10$ $P < 95\%$

Note: $F_{critical}$ at 5% in tail = 3.25.

6) You calculate the following models (1) and (2) for casinos, Casinos 1 and 2, by OLS regression.

Coin In_t 1 = $\beta_0 + \beta_1 \text{Promo}_t + u_t$ (1)

Coin In_t 2 = $\beta_0 + \beta_1 \text{Promo}_t + u_t$ (2)

Given the regression output below, test the hypothesis $H_0: \beta_1 \text{ for 1} > \beta_1 \text{ for 2}$? Use $\alpha = 5\%$ and $d.f. \cong 198$ for your test.

OLS for 1

	Coef	S.E. of Coef
β_0	10	5
β_1	25	4

OLS for 2

	Coef	S.E. of Coef
β_0	10	5
β_1	10	3

$t_{calculated} = \frac{OLS \beta_1 \text{ for model 1} - OLS \beta_1 \text{ for model 2}}{\sqrt{SE_1^2 + SE_2^2}} = \frac{25 - 10}{\sqrt{16 + 9}} = 3$

$t_{critical} = 1.645$

Decision rule: If $t_{calc} > t_{critical}$ then do not reject H_0 .

Decision: Do not reject H_0 !

7. If $Y = \beta_0 + \beta_1 X + \beta_2 W + \varepsilon$ then what is σ_Y^2 given the information below? Assume that X, W, and ε are independent.

$\beta_0 = 5, \beta_1 = 2, \beta_2 = 1, \sigma_X^2 = 1, \sigma_W^2 = 4, \sigma_\varepsilon^2 = 2$

$$\sigma_Y^2 = \beta_1^2 \sigma_X^2 + \beta_2^2 \sigma_W^2 + \sigma_\varepsilon^2 = 2^2(1) + 1^2(4) + 2 = 4 + 4 + 2 = 10$$